1 Context-free grammars

Exercice 1. Let us consider the following grammar on the alphabet $\{x, y, +, -, *\}$:

$$E \to +EE \mid *EE \mid -EE \mid x \mid y$$

1. Find the leftmost and the rightmost derivations for the string $+ * -xyxy$.
2. Show that this grammar in unambiguous.
3. Find a PDA for this grammar.

Exercice 2.
Using the pumping lemma show that the following languages are not CFG :

1. $\mathcal{L}_0 = \{a^i b^j c^k \mid i < j < k\}$
2. $\mathcal{L}_1 = \{a^n b^n c^m \mid n \leq m \leq 2n\}$
3. $\mathcal{L}_2 = \{a^{2^n} \mid n \in \mathbb{N}\}$
4. $\mathcal{L}_3 = \{a^{n^2} \mid n \in \mathbb{N}\}$

2 Chomsky Normal Form (CNF)

Definition 1. *We recall that a grammar in a CNF when all the production rules are :*

$$A \to BC \quad or \quad A \to a \quad or \quad S \to \epsilon$$

with $B \neq S$ and $C \neq S$ where S is the start symbol.

Exercice 3. Let $G = (\Sigma, N, P, S)$ be a CFG. We suppose that S never appears in the right side of a production rule, how can you eliminate all the production rules of the form $A \to \epsilon$ (except eventually $S \to \epsilon$) from G ?

Exercice 4. Devise a method to transform production rules into CNF form, for the following shapes of production rules (we suppose that S does not appear) :

1. $A \to bC$
2. $A \to Bc$
3. $A \to bc$
4. $A \to BCD$
5. $A \to \alpha_1\alpha_2\alpha_3$ with $\alpha_i \in \Sigma \cup N$
6. $A \to \alpha_1 \dots \alpha_p$ with $p \geq 3$
7. $A \to B$

Exercice 5. Devise a CNF equivalent to the following grammar :

$$\begin{array}{rcrcl} S & \to & CSC & \mid & aB \\ C & \to & B & \mid & S \\ B & \to & b & \mid & \epsilon \end{array}$$

Exercice 6. Devise a polynomial time algorithm (in the size of word and the grammar) recognizing when a word belongs to a CNF grammar.