

Find a deterministic tree automata for the following languages:

Exercise 1. The words over the alphabet $\{a, b\}$ containing the factor aab or $aaab$.

Exercise 2. The words over the alphabet $\{a, b\}$ containing an even number of a and an odd number of b .

Exercise 3. The words over the alphabet $\{a\}$ with a length that is a multiple of 3.

Exercise 4. For all $d \in \mathbb{N}$, the words over the alphabet $\{a\}$ with a length that is a multiple of d .

Exercise 5. The binary representation of even integers. Integer are positive and the numbers are given in the big-endian order (i.e. natural order: 101010 is 42).

Exercise 6. For all $d \in \mathbb{N}$, the binary representation of multiples of d .

Exercise 7. For all $d \in \mathbb{N}$ the binary representation of numbers of the numbers $c + k \times d$ for $k \in \mathbb{N}$.

Given a regular language \mathcal{L} prove that the following languages are also regular:

Exercise 8. $Init(\mathcal{L}) = \{u \mid \exists v : uv \in \mathcal{L}\}$

Exercise 9. $Min(\mathcal{L}) = \{w \in \mathcal{L} \mid \nexists u \in \mathcal{L} : u \text{ strict prefix of } w\}$

Exercise 10. $Max(\mathcal{L}) = \{w \in \mathcal{L} \mid wu \in \mathcal{L} \Rightarrow u = \epsilon\}$

Exercise 11. $Cycle(\mathcal{L}) = \{uv \mid vu \in \mathcal{L}\}$

Exercise 12. $\frac{1}{2}\mathcal{L} = \{u \mid \exists v : uv \in \mathcal{L} \text{ and } |v| = |u|\}$