Translation from μ-calculus WS2S to tree automata

Internship

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XML : : organizing data

- Data is organized in a tree form
- Nodes have a label and some attributes
- XML is universal and because many different applications use XML we give the XML files a type. This type describes some rules the file comply with.

```
<addressBook>
    <entry>
        <name>Napoleon</name>
        <email>Napo@leon.com</email>
    </entry>
    <entry>
        <name>Emer Gency</name>
        <phoneNumber>0118 999 881 999 119 7253</phoneNumbe
    </entry>
    <entry>
        <name>Spiderman</name>
    </entry>
</addressBook>
```

A finite tree automaton (NFTA) over $\mathcal{F}$ is a tuple $\mathcal{A} = (\mathcal{Q}, \mathcal{F}, \mathcal{Q}_f, \Delta)$ where :

- $\mathcal{Q}$ is a set of states
- $\mathcal{Q}_f \subset \mathcal{Q}$ is a set of final states
- Δ is a set of transition rules in the form : $f(q_1(x_1), \ldots, q_n(x_n)) \rightarrow q(f(x_1, \ldots, x_n))$, where $n \geq 0$, $f \in \mathcal{F}_n, q, q_1, \ldots, q_n \in \mathcal{Q}$, $x_1, \ldots, x_n \in \mathcal{X}$.

How to recognize a tree with a label "a"?

Properties

Tree automata share with automata on words most of their properties :

- complementation, union, intersection
- determinization, minimization
- equivalence with regular tree language
- equivalence with monadic second order logic

- $\varphi = \varphi_1 \vee \varphi_2$
- $\varphi = \varphi_1 \wedge \varphi_2$
- $\varphi = \neg\varphi'$
- $\varphi = \langle a \rangle \varphi'$ where $a \in \{1, 2, \bar{1}, \bar{2}\}$. We have $\bar{\bar{a}} = a$.
- $\varphi = \mu \overline{X_i = \varphi'_i}$ *in* ψ
- X where X is a variable.
- $\varphi = \top$
- $\varphi = \sigma, \sigma \in \mathcal{A} \cup \mathcal{B}$

- $[\![\varphi_1 \wedge \varphi_2]\!]_V = [\![\varphi_1]\!]_V \cap [\![\varphi_2]\!]_V$.
- $[\![\varphi_1 \vee \varphi_2]\!]_V = [\![\varphi_1]\!]_V \cup [\![\varphi_2]\!]_V$.
- $[\![\neg\varphi]\!]_V = \mathcal{F} \setminus [\![\varphi]\!]_V$.
- $[\![\langle a \rangle \varphi]\!]_V = \{\mathcal{T}\langle \bar{a} \rangle \mid \mathcal{T} \in [\![\varphi]\!]_V \wedge \mathcal{T}\langle \bar{a} \rangle \text{ defined } \}$
- $[\![\mu\overline{X_i = \varphi_i} \textit{ in } \psi]\!]_V = \text{let } T_i = \{ \bigcap_{T_i \subset \mathcal{F}} T_i \mid [\![\varphi_i]\!]_{V\{\overline{T_i/X_i}\}} \subset T_i\}$ in $[\![\psi]\!]_{V\{T_i/X_i\}}$
- If $\varphi = \sigma$ where $\sigma \in \mathcal{A}$, $[\![\varphi]\!]_V = \{\mathcal{T} \in \mathcal{F}, \mathcal{L}(\mathcal{T}) = \sigma\}$.
- If $\varphi = \sigma$ where $\sigma \in \mathcal{B}$, $[\![\varphi]\!]_V = \{\mathcal{T} \in \mathcal{F}, \sigma \in \mathcal{S}(\mathcal{T})\}$.

Definitions

- Expanding of fix point
- Modality-free variables
- Modality paths and cycle-free formulas
- Focused trees
- $\mathcal{L}ean$
- automata over trees with attributes

- Q the set of types
- For any tuple of types x, y, z, any label $c \in \mathcal{A}$ and any set of attributes $\mathcal{O} \subset \mathcal{B}$ we have :
  - If $\Phi_c(x) \Rightarrow \langle 1 \rangle \top \wedge \langle 2 \rangle \top$ then δ contains $\begin{smallmatrix} & c & \\ y & & z \end{smallmatrix} \rightarrow x$ iff $\Delta_1(x, y) \wedge \Delta_2(x, z) \wedge \mathcal{L}(x) = c \wedge \mathcal{O} = \mathcal{S}(x)$
  - If $\Phi_c(x) \Rightarrow \neg \langle 1 \rangle \top \wedge \langle 2 \rangle \top$ then δ contains $\begin{smallmatrix} & c & \\ y & & \end{smallmatrix} \rightarrow x$ iff $\Delta_1(x, y) \wedge \mathcal{L}(x) = c \wedge \mathcal{O} = \mathcal{S}(x)$
  - If $\Phi_c(x) \Rightarrow \langle 1 \rangle \top \wedge \neg \langle 2 \rangle \top$ then δ contains $\begin{smallmatrix} & c & \\ & & z \end{smallmatrix} \rightarrow x$ iff $\Delta_2(x, z) \wedge \mathcal{L}(x) = c \wedge \mathcal{O} = \mathcal{S}(x)$
  - If $\Phi_c(x) \Rightarrow \neg \langle 1 \rangle \top \wedge \neg \langle 2 \rangle \top$ then δ contains $\begin{smallmatrix} & c & \\ & & \end{smallmatrix} \rightarrow x$ iff $\mathcal{L}(x) = c \wedge \mathcal{O} = \mathcal{S}(x)$
- $Q_f = \{q \in Q \ / \ \Phi_c(q) \Rightarrow \mu X = \xi \vee \langle 1 \rangle X \vee \langle 2 \rangle X \text{ in } X \wedge \neg \langle \bar{1} \rangle \top \wedge \neg \langle \bar{2} \rangle \top\}$.